[redacted]

VET. NOV. TES TA- MEN TUM

DEI SUB NUMINE VIGET

UNCLASSIFIED
[redacted]

Clas... anged to

by authority of L. Spitzer, Jr.

by C R Shock 7/16/53

PROJECT MATTERHORN

Contract AT(30-1) – 1238 with the
U S Atomic Energy Commission

PRINCETON UNIVERSITY

PRINCETON, NEW JERSEY

SECRET

CONFIDENTIAL

This document consists of 35 Page(s)
No. 13 of 20 Copies, Series A

PROJECT MATTERHORN
Forrestal Research Center
Princeton University
Princeton, New Jersey

UNCLASSIFIED

CONFIDENTIAL

Classification changed to CONFIDENTIAL
by authority of L Spitzer, Jr
by CRShack 7/16/53

July 23, 1951

PM-S-1
NYO-993

CLASSIFICATION CANCELLED
For The Atomic Energy Commission
by the Declassification Officer
per U.S. AEC [signature]
by E G Aubin[?]
date NOV 15 1958

A PROPOSED STELLARATOR

Distribution, Series A

Cy #	to
1-4	Div. of Res. AEC, Wash
5	NY Operations Office
6	JR Oppenheimer
7	Institute for Nuclear Studies, Chicago
8-10	Los Alamos Lab
11-12	U of Cal Radiation Lab.
13-20	Project Matterhorn

Report written by:

Lyman Spitzer, Jr.

RESTRICTED DATA
This document contains restricted data as defined in the Atomic Energy Act of 1946.

UNCLASSIFIED

SECRET

CONFIDENTIAL

UNCLASSIFIED

This report is essentially a reprint of a paper from the Department of Astronomy, Princeton University, dated May 12, 1951, and prepared independently of Project Matterhorn or any other government-supported enterprise. Since it is planned that Project Matterhorn will devote considerable effort to analysis of the Stellarator, this paper is being duplicated and distributed by the Project.

UNCLASSIFIED

A Proposed Stellarator

Abstract

This report analyses the possible performance of a device designed to obtain power from the thermonuclear reactions between deuterium and either deuterium or tritium, It appears from this theoretical study that a ataady-state generator or "Stellarator" may be feasible. Such a thermonuclear reactor would find important uses both as a power source and as a neutron generator.

I. Introduction

Controlled nuclear fission, in addition to its enormous military importance, has opened up important new possibilities both in power generation and in the use of very large quantities of neutrons. This technique suffers from the disadvantage that uranium, the basic raw material in fission, is scarce and expensive. In principle, reactions between the hydrogen isotopes, deuterium and tritium, can also release atomic power and generate neutrons. While tritium must be produced artificially, deuterium occurs

UNCLASSIFIED CONFIDENTIAL

in essentially unlimited quantities in the ocean, and is potentially an enormous source of useful power if means can be found to tap this energy.

In addition to its importance as a source of power, reactions between deuterium nuclei are important as a generator of tritium. There seems little question that a deuterium-tritium bomb could be built with vaet destructive power. Any device which might produce considerable quantities of tritium is therefore of great military interest.

In the present report, a possible method for tapping the energy of deuterium is presented, and the general outlines of a specific device for this purpose are discussed. The theoretical study is incomplete, but the preliminary results indicate that the device propo'sed here might well be capable of generating atomic power and tritium on a very large scale. First the general requirements for liberation of this atomic energy are discussed below. Next the reactions proceeding under certain idealized conditions are analyzed. Finally, the modifications necessary to produce a practical device are presented and discussed.

UNCLASSIFIED CONFIDENTIAL

II. General Requirements

Two light nuclei can liberate energy only if they approach each other sufficiently closely to interact, i.e., within about 10^{-12} cm. Because of the strong electrostatic repulsion between nuclei, this means that their relative velocities must be very high. In this respect, thermonuclear reactions between light elements differ from the fission of uranium which may proceed at low atomic velocities. The velocities of nuclei may be measured in terms of the temperature T. Virtually no thermonuclear reactions will take place if T is less than 1,000,000° Kelvin, and to achieve an appreciable reaction rate T must be about 100,000,000°K or even more. Since this result refers only to the velocities of the nuclei, T may be called a "kinetic temperature" for these nuclei.

One might suppose that a reaction could be obtained if two directed beams of nuclei were passing through each other, with little random motion. In general this is not possible, since the probability of a deflection by electrostatic forces is much greater than the probability of a nuclear reaction. When two moving particles interact with each other, we define as the "collision parameter" b the distance of closest approach which would result if there were no force

between the two particles. In a collision between two hydrogen nuclei, each with a charge e, the deflection of each nucleus will be about 90° or more provided that their mutual electrostatic energy at the distance b, numerically equal to e^2/b, is as great as or greater than the kinetic energy 3kT/2, where k is the usual gas constant. If T equals 100,000,000°, these two energies are equal when b is 10^{-11} cm, some ten times the nuclear radius. On this simple picture, a nucleus will be deflected through 90° or more a hundred times before it reacts with another nucleus, and it is evident that the velocities of the nuclei will be wholly random. Actually, the cumulative effect of more distant encounters is about a hundred times as great as the effect of single close encounters, thus reinforcing the result. We conclude that random velocities corresponding to a temperature of 100,000,000° or more are an essential requirement for releasing the energy of deuterium, tritium, or any of the light elements.

A second requiremsnt is that the density of the interacting gases be kept low. This is necessary if the pressure of the gas is to be sufficiently low to avoid an explosion. If n is the number of particles per cubic centimeter, the pressure in atmospheres is

about $10^{-6}nkT$, where k is again the gas constant. If the pressure is not to exceed 10 atmospheres, and T is 10^8 degrees K, then n must not exceed 10^{15} particles per cubic centimeter, less than the particle density of ordinary air, by about 1/10,000. For intermittent operation it might be possible to use higher densities in a smaller region, which would expand into a bigger container, but for steady operation these lower densities are an essential requirement.

The third requirement is the most difficult to fulfill. At the low densities required for practical steady operation, an ion or electron will, spend most of its time moving freely; in other words, its free path between collisions is very long. For example, if the particle density n is 10^{14} per centimeter, a deuteron will travel 300 kilometers before it is deflected 90° or more by a collision with another deuteron. Consideration of cumulative small deflections decreases this mean free path to 3 kilometers. If the deuteron collides during this time with a material wall, whose temperature cannot exceed some thousand degrees at most, it will lose a large fraction of its hundred-million-degree energy. Evidently, to maintain so high a temperature, the nuclei must be prevented from hitting the wall. If the ions move in straight lines, this requirement would result in a vessel whose dimensions were

many kilometers, which is clearly impractical.

To keep the ions from hitting the wall, some type of force is required that will act at a distance. Gravitational forces are too small. Electrical forces act oppositely on positive ions and electrons and cannot simultaneously confine both types of particles. Since electrons must always accompany positive ions in equal numbers, to avert the production of colossal electrical fields, and since the electrons will tend to have the same energy as the positive nuclei, both types of particle must be confined.

A magnetic field seems to offer the only promise of confining both electrons and positive ions within a small volume, preventing them from colliding with the walls. In the presence of a strong magnetic field, a charged particle simply circles about the field direction, or lines of force, and moves only slowly across the field. For a deuteron moving at 1000 kilometers per second, corresponding to a kinetic temperature of about 10^8 degrees, the radius of curvature in a magnetic field of 20,000 gauss is closely equal to 1 centimeter, and for an electron is about 0.02 centimeters. If a strong current is run around a tube, and the tube is bent around so that the two ends are joined to form a

continuous circuit, the magnetic lines of force will stay inside the tube, and charged particles will tend to follow these lines of force, without; encountering the walls. Confinement of an ionized gas within a tuba of this sort by magnetic fields may be taken as a third requirement of any device liberating thermonuclear energy in a controlled manner.

In the following sections the performance of a device conforming to these requirements is investigated. Since the proposed system generates power and neutrons by reactions similar to those occurring in stars, the device analyzed below is called a "Stellarator". The discussion is necessarily somewhat mathematical, and a summary of the physical characteristics of a proposed Stellarator is given in the last section,

III. Power Equilibrium in an Ideal Stellarator

First we consider the performance of a Stellarator in which the magnetic field is so strong that the positive ions and electrons are completely confined within the tuba, and no collisions with the walls occur. The energy required to produce such a magnetic field will also be neglected. All difficulties connected with the magnetic field will be discussed subsequently.

We are concerned here with the rate at which power is generated by thermonuclear reactions and the rate at which heat is lost by radiation. Evidently the power generated must considerably exceed the power radiated if we are to have a sufficient margin of safety when the imperfections of the magnetic field are considered. Let us consider a small segment of the tube, only 1 centimeter long; the cross section of the segment is assumed to be a circle, with a radius of a centimeters. The number of particles per cubic centimeter will again be equal to n, and their kinetic temperature to T. We shall assume that the positive ions have an average charge Ze, and that the number of electrons n_e equals Z times the number of positive ions, n_i, and that the sum $n_e + n_i$ equals n. Let N be the number of nuclear reactions taking place in each cubic centimeter per second, E the energy liberated per reaction, and P_N the nuclear power generated per centimeter length of tube. Evidently EN is the power generated per cubic centimeter and

$$P_N = \pi a^2 EN, \qquad (1)$$

since πa^2 is the area of the tube cross section.

The probability that one ion collides with

another va: es as n_1, the density of positive ions. per cm^3, a d N, the total number of reactions per cubic centimeter per second may therefore be written

$$N = \alpha \, n_1^{\,2}, \tag{2}$$

where α is some constant that depends on the temperature. In terms of the reaction cross section σ and the Maxwellian distribut on function P(v) for the relative velocities of the ions, it is evident that

$$\alpha = \xi_1 \xi_2 \int_0^\infty v \, \sigma \, P(v) \, dv \, . \tag{3}$$

If 1 and 2 are used to designate the two types of interacting posit: ve ions, ξ_1 is the fraction of the ions which are of type 1, while ξ_2 is the corresponding fraction for type 2. If onl; one type o: positive ion i s present, the product $\xi_1 \xi_2$ may b(set equal to one-half.

A8 we shell see later, the total number of particles per cubic centimeter, which we denote by n, bears a critical relationship to the strength H of the magnetic field, and we @hall define x by the equation

$$nkT = \frac{x \, H^2}{8\pi}$$

e ...

Since the average Ionic charge is Z, it is evident that

$$n_1 = \frac{n}{1+Z} \quad . \qquad (5)$$

Combining equations (1), (2), (4) and (5), we obtain

$$P_N = \frac{\alpha x^2 a^2 E H^4}{64\pi (1+Z)^2 k^2 T^2} \quad . \qquad (6)$$

For the deuteron-triton reaction at 10^8 degrees, α is about 2×10^{-17} cm^3/sec, and E is 17.6 Mev., or 2.8×10^{-5} ergs; this value of E includes only the energy directly liberated, and neglects the additional energy which might be obtained from the liberated neutrons in subsequent reactions outside the Stellarator. If we let H equal 2×10^4 gauss, x equal 1/2, a equal 50 centimeters, Z equal 1, and T equal 10^8 degrees, then for these standard conditions we have

$$P_N = 37{,}000 \text{ watts/cm} \quad . \qquad (7)$$

Four-fifths of this energy would be carried away by the neutrons produced, with some 7 kilowatts per centimeter available as kinetic energy of the alpha particles, which would mostly be retained in the Stellarator. In this standard case, n is 5.8×10^{14} per cubic centimeter, and n_1 is half of this.

The radiation of energy by the ions and electrons must also be considered. The electrons will be accelerated as they pass by the ions, emitting a pulse of

electromagnetic radiation. The radiated power P_R may be computed from quantum mechanics; one obtains the equation

$$P_R = \frac{2^5 \cancel{\pi^2} a^2 E_0 Z^3}{\cancel{2\pi^3}(1+Z)^2} \cdot \frac{e^2 h}{3 m_e^2 c^3} \cdot \left(\frac{kT}{2\pi m_e}\right)^{1/2} \frac{x^2 H^4}{64\pi^2 k^2 T^2} , \qquad (8)$$

Too small by a factor 2!

where E_o is the ionization energy of the H atom, m_e is the mass of the electron, h is Planck's constant, and other symbols have their usual meanings. For the standard conditions specified above we find

$$P_R = 510 \text{ watts/cm} . \qquad (9)$$

This radiative loss is clearly negligible compared with P_N. Generation of radio waves by spiralling electrons is also negligible, since the radiated energy is absorbed by one electron as fast as it is produced by another, and very little escapes.

For the deuteron-deuteron reaction, α is less by a factor of about a hundred, and P_N will be comparable with P_R. However, a moderate increase of T will increase α about as T^2, keeping P_N nearly constant, while P_R will decrease.

IV. Performance of Infinite Cylindrical Stellarator

In an actual Stellarator, losses of energy will arise from the current needed to generate the magnetic field and from collisions between the charged particles and the walls

of the Stellarator. Such losses are considered here, together with the related problems of electric fields and stability.

First we consider the electrical power P_H per centimeter length of tube required to generate the magnetic field We assume that around the hot gas a current of density j amperes per square centimeter flows. This current flows in circles centered at the tube axis, and in planes perpendicular to the tube axis. Let the region in which this current flows be a cylindrical shell, of inner radius r_1, and of outer radius r_2. Then inside r_1 the field strength H will be given by the equation

$$H = \frac{4\pi j}{10}(r_2 - r_1) \quad . \qquad (10)$$

The power dissipation P_H per linear centimeter, which we shall measure in watts, becomes

$$P_H = (\pi r_2^2 - \pi r_1^2)\frac{j^2}{\sigma} = \frac{25}{4\pi}\frac{r_2 + r_1}{r_2 - r_1}\frac{H^2}{\sigma} , \qquad (11)$$

where σ is the conductivity in the material through which the current flows.

Let us consider first a system in which these currents flow in copper coils surrounding the tube containing the reacting gas. We shall denote this as System A. The conductivity of copper at 100° C is 4.4×10^5 ohm^{-1} cm^{-1}. We divide this value by two, on the assumption that half the

physical volume is occupied by cooling liquid, insulation, etc. If we let r_2/r_1 equal 2, then for a field of 20,000 gauss we have

$$P_H = 11,000 \text{ watts/cm} \qquad (12)$$

This is large, but subatantially smaller than P_N for the deuteron-tritium reaction.

In principle, the magnet coils could be cooled to about -190°C, near the temperature of liquid air. The conductivity at these low temperatures is about ten times the assumed value, To cool this much air, however, about as much electrical power would be needed as to keep the current in the magnets going. In this case, then we would have

$$P_H = 2,200 \text{ watts/cm} . \qquad (13)$$

For the Stellarator of dimensions specified above, with r_2/r_1 equal to 2, the power dissipated would be only about 0.1 watts/cm3, and the cooling problem should not be difficult. Since P_N for the deuteron-deuteron reaction is less than that for the deuteron-tritium reaction by 4 factor of about a hundred, a reduction of P_H by this method might be necessary for a working deuteron-deuteron Stellarator whose size and magnetic fields were kept to minimum values. If liquid hydrogen could be used as a coolant, P_H could be reduced by another factor of ten,

There is another way in which P_H might be

decreased. The conductivity of ionized hydrogen gas at 10^8 degrees is 3×10^7 ohm^{-1} cm^{-1}, about the same as copper at the temperature of liquid hydrogen. If the current can be made to flow around inside the tube containing the reacting gas, P_H can be reduced to a negligible value. We shall outline a possible system, which we call System B, which takes advantage of the high conductivity of an ionized gas.

In System B, a strong current is passed through external coils for a brief moment, - about a second or less - generating a strong magnetic field. This magnetic field is produced when the temperature of the gas is relatively low, - less than a million degrees K, - and the conductivity is therefore sufficiently low so that the magnetic field permeates the gas freely. When this magnetic field is at its height, the temperature of the gas is increased to its operating value, about 10^8 degrees, by a glow discharge of some sort. This increases the conductivity of the gas enormously. The external current is then turned off, and the magnetic field inside the gas decays very slowly, owing to the high conductivity, requiring about two minutes to drop to 1/e of its initial value, for a tubs of radius 50 cm. At a temperature of 4×10^8 degrees (an increase needed to make use of the deuteron-deuteron reaction) the conductivity is eight times greater and the corresponding decay time is about 15 minutes.

Among several problems posed by System B is that the magnetic field is produced in the high-temperature region, and falls off near the outer boundary of this region, where a magnetic field is most needed to prevent the hot gas from diffusing outward. In addition, since the total pressure must be constant throughnut this system, nkT will be least on the axis of the tube, where $H^2/8\pi$ is greatest, thus decreasing further the density in the hottest region, where the nuclear reactions take place. Rather detailed computations would be needed to predict the probable performance of System B, and chief attention is therefore devoted here to System A, whose operation can be examined with somewhat less uncertainty.

Next we consider diffusion losses at the wall. The flow of heat across the tube should be determined by the solution of the appropriate differential equation, taking into account the variation of heat conductivity with H. However, in this preliminary discussion an approximate treatment will suffice. In the absence of collisions each ion will spiral about the lines of force in a circle of radius ρ. After each collision, the ion will be in a new circle, whose center may be shifted a distance ρ, on the average, from its former

position. If τ is the time between collisions, the center of the circle in which the ion moves will therefore wander a distance ρ in the time τ. As in familiar Brownian motion, after a time t the center of this circle will be displaced a distance L, on the average, where

$$L^2 = \rho^2 \frac{t}{\tau} . \qquad (14)$$

In the present instance the ion will collide with the wall, on the average, when L equals a. Hence during each interval of time $\tau (a/\rho)^2$ the entire kinetic energy of the ions and electrons in the tube will be lost to the wall. The wall loss P_W per centimeter length of tube then becomes

$$P_W = \frac{3\pi a^2 n_1 kT}{2(a/\rho)^2 \tau} \qquad (15)$$

The value of the time τ between collisions may be taken from Chandrasekhar. For ions of the root mean square velocity, the time required for the cumulative deflection in many encounters to reach 90° is

$$\tau = \frac{0.063\, m_1{}^2 v^3}{n_1\, e^4 \log B} , \qquad (16)$$

where log B is a quantity which for these conditions equals about 20. Under the standard conditions assumed here, we find for deuterons

$$\tau = 3.3 \times 10^{-3} \text{ sec} . \qquad (17)$$

The radius of gyration ρ is given by the usual formula

$$\rho = \frac{m v c}{e H} . \qquad (18)$$

For deuterons under standard conditions, the root mean square velocity perpendicular to H is 0.9×10^8 cm/sec and

$$\rho = 0.9 \text{ cm} . \qquad (19)$$

Thus a/ρ is about 55 for the standard conditions and particles reach the wall after a time interval of $3 \times 10^3 \tau$, on the average, or about 10 seconds. For electrons ρ is only 0.015 cm.

If we combine these various results, P_W becomes

$$P_W = \frac{0.91 \, x^2 \, m_1^{\frac{1}{2}} \, e^2 \, c^2 \, H^2}{(1+Z)^2 \, (KT)^{3/2}} \qquad (20)$$

For the standard condition, we find

$$P_W = 530 \text{ watts/cm} , \qquad (21)$$

about equal to P_R. Evidently the wall losses are small, in the ideal case considered, compared to P_H in System A. If T were increased by a factor of about 2, with a corresponding decrease of n, P_N would be unaffected while P_W and P_R would each decrease by a factor of about 3 and would be less than one percent of P_N.

This analysis is very simplified and neglects

a number of major problems. Firstly, the heat flow to the walls, determined from more detailed considerations, may differ materially from the value found above. Secondly, the possible presence of electrical fields, both steady and transient must be considered. Thirdly, the general stability of operation must be discussed. Each of these is considered separately, and some modifications suggested in the proposed device.

The flow of heat outwards is governed by the familiar equation of heat conductivity. If a steady state is set up, one might suppose that the temperature would decrease steadily from a high value at the center of the tube to the relatively very low value of perhaps 1000° K at the walls. In the steady state, however, the pressure will be uniform throughout the tube, and the density will vary inversely as the temperature, with a particle density at the walls 10^5 times that at the center. With increasing density the thermal conductivity across the magnetic field increases, and most of the temperature drop will occur relatively close to the center of the tube. As a result, the effective tube radius which must be used in all the previous computations may be substantially reduced from the previous value. In System B this would not be serious, since, for the deuterium-tritium reaction,

a smaller effective value of a would still have P_N much greater than the other losses. In System A, however, the value of a cannot be materially decreased if P_N is to exceed P_H, and any large increase in the actual outer radius of the tube would increase the amount of copper; even for the standard Stellarator as specified above, if r_1 is set equal to 50 cm, which leaves no space for the lithium needed to absorb the neutrons, ten tons of copper would be required for each meter length of tube.

To circumvent this difficulty with System A, a modification in this system is proposed. The basic purpose of this modification is to take the material out of the tube before it reaches the walls, thus eliminating any drop of temperature in the tube. To effect this purpose, auxiliary tubes would be positioned in pairs at the wall of the main tube, each tube encircled by current-bearing coils. These tubes, each with its own magnetic field, would be so positioned as to bend the lines of force near the wall into the auxiliary tubes, and sway from the Stellarator. To avoid the necessity of very strong magnetic Fields in these auxiliary tubes, these tubes would best be placed in a gap in the main magnetic field, where the main field will be somewhat weakened, especially near the tube walls. Iron connection pieces might be

provided to carry away the magnetic lines of force travelling through the magnet coils, leaving more space into which the lines of force near the walls might expand, with resultant weakening. The set-up envisaged is shown in Figure 2 at the end of this paper.

Conditions in the auxiliary tubes would be planned to diminish as much as possible the chance that ione and electrons would leak back into the main tube. Along these auxiliary tubes the cross sections would increase, and H diminish, with increasing distance from the main tubs, and finally the ions and electrons would be stopped at some material barrier and neutralized. Since a spiralling ion behaves diamagnetically, it tends to be re-gelled from regions of strong magnetic field, and thus few ions would leak back. The neutral atoms can then be pumped out, the gas compressed, possibly some "burnt gas" taken out and replaced by fresh reactants, and the gas then Injected back into the main tube in the form of jets.

In this proposed System A the ions and electrons drift toward the walls but are taken out of the tube before they reach it, the "wall losses" all occurring at the end8 of the auxiliary tubes. Since there is now no flow of heat, divorced from the transport of atoms, the temperature should be very nearly uniform over the tube.

The density will fall off with increasing distance from the axis, since a pressure gradient is necessary to maintain an outwards drift of ions and electrons. Since the sum of magnetic and material pressures must be uniform throughout; the tube $H^2/8\pi$ will increase radially outwards as nkT diminishes, and nkT at the axis cannot exceed $H^2/8\pi$ at the walls. From preliminary calculations it would appear that the outward drift results entirely from electron-positive ion encounters, is the same for both electron and positive ions, and is less by an order of magnitude than the value found from the interactions between positive ions considered above. Hence P_W may be essentially negligible in System A.

Conditions in the jets of gas shot back into the Stellarator must also be considered. In such a jet the initial density will be much higher than in the Stellarator, but the temperature much lower. The material will all be ionized before it has gone far, but at least a millisecond will be required for the temperature to rise to 10^8 degrees. Since a jet velocity of over a kilometer per second should be feasible, the jet should be able to travel across the Stellarator before it dissipates. The magnetic field will not stop the mass motion of the jet, since polarization charges on the sides of the jet will produce an electrical field which counterbalances the

effect of the magnetic field. As we shall see immediately below, these jets play an important part in carrying current across the magnetic field and thus help to neutralize space-charge effects.

We proceed now to discuss the effect of electrical fields of various types. When a strong magnetic field is present in an ionized gas, an electrical field perpendicular to H produces primarily not an electric current, but a drift motion perpendicular to both E and H, the same for both electrons and positive ions. Conversely, a force F perpendicular to H produces no motion in that direction, but a current perpendicular to F and H.

In the Stellarators discussed here, electrical fields perpendicular to H can be produced in various ways. In System A some of the positively charged reaction products (He^4 nuclei, for example) will escape from the main tube, leaving the gas with a negative electrostatic charge; the resultant field produces a rotation of the entire mass of gas. Since little or no contact with the walls is established, there is no shearing stress to slow down this rotation. The jets of gas shot back into the Stellarator provide a simple means for reducing the electrostatic field, since the cool material in the jet will carry an appreciable radial current before the

magnetic force on the current accelerates the material until it shares in the general rotation about the axis.

In System B jets can again be provided, taking the gas of neutral atoms out at the walls, compressing it and shooting it back in. Some method for taking the "burnt" gas out and injecting fresh reactant is required in any case, and the jets will help to reduce the electrostatic fields.

Transitory electrical fields may be important in either system. Since an electrical field does not produce a current directly, but only indirectly through dynamical effects, a field due to a local charge distribution will not rapidly disappear. The enormous conductivity along the lines of force will maintain a constant potential all along any one line of force, but on adjacent lines a different potential may be found. Consider a length L of the tube, and consider a small cylinder along the tube, with a radius of 2 centimeters and a cross section of 12 square centimeters. Since the radius of gyration of a positive ion is about 1 centimeter under the standard conditions, most ions inside this cylinder will remain inside during the effective time τ between collisions. If the density of ions is $3 \times 10^{14}/cm^3$, the total number of ions in this tube will be 4×10^{15} L.

Statistical fluctuations will produce deviations of about the square root of this quantity, or $6 \times 10^7 L^{\frac{1}{2}}$, yielding a charge fluctuation of about $3 \times 10^{-2} L^{\frac{1}{2}}$ e.s.u. The field E resulting from this charge fluctuation will be $1.5 \times 10^{-2}/L^{\frac{1}{2}}$ e.s.u. at the boundary of the cylinder. The velocity resulting is c E/H, or $2 \times 10^4/L^{\frac{1}{2}}$ cm/sec. For an infinite cylinder there is no effect, and even if L is only 100 cm, the velocity is not serious. However, fluctuations much larger than the average might produce material velocities great enough to provide a serious source of dissipation. The jets proposed above might provide conducting paths which could make possible the neutralization of potential differences between different lines of force.

Finally we must consider the stability of operation of a Stellarator. It must be assumed that the neutrons escape from the gas and that the losses of energy by radiation and at the walls are exactly counterbalanced by energy gained from the charged reaction products, - He^4 in the case of the deuteron-triton reaction. We have seen that P_N vastly exceeds the wall losses for this deuteron-triton reaction, and that even a small fraction of the energy available in He^4 recoil nuclei would suffice to keep the reaction going. Since ρ for a He^4 nucleus with 1 Mev of energy is only 12 centimeters it is evident

that most He4 recoil nuclei would in fact; remain in the tube, spiralling about the line of force until they gradually slowed down. To maintain a balance between input and output in System A, the density could be increased by nearly a faotor of 2 over standard' conditions, making nkT at the center nearly equal to the value of $H^2/8\pi$ at the wall, and reducing H at the center to a very low value. This would increase the wall losses. Alternatively, P_R could be increased by adding ions of higher nuclear charge. In Pact, the presence in the tube of He^4, the end product of the deuteron-triton reaction, would increase P_R materially, and would also decrease P_N somewhat. Near the equilibrium point System A would be stable, provided the nuclear coefficient α increased less rapidly than T^2. In such a case, an increase of T decreases the density n, and decreases the reaotion rate, tending to reduce T. Similarly, a decrease of T increases the reaction rate. System B would probably also be stable, provided that constant pressure is maintained in the Stellarator, although the possible variation of H with T complicates the problem.

IV. Performance of Finite, Curved Stellarator

To operate a finite Stellarator, the two ends of a cylindrical Stellarator must somehow be terminated without loss of the energetic particles. The simplest

means to effect this is to bend the tube into a circle and join the ends together, forming a torus, or doughnut-shaped Stellarator. Most of the characteristics of the cylinder Stellarator are also shared by such a toroidal Stellarator. In addition, new difficulties appear resulting from the curvature of the magnetic field.

When a magnetic field is not completely uniform, charged particles will not simply spiral about the lines of force but will also show a drift velocity across the lines of force. For example, if the magnetic field H in the z direction increases in the x direction, an ion of charge e and mass m will show a drift velocity v_y along the y axis numerically equal to

$$y = \frac{m u^2 c}{2eH^2} \frac{dH}{dx} , \qquad (22)$$

where u is the velocity perpendicular to H. The positive ions will drift in the opposite direction from the electrons. Similarly, if a particle is moving along the lines of force (in the z direction) with a velocity w, and these lines bend in the z-x plane with a radius A, electrons will again drift in the y direction with the velocity

$$v_y = \frac{m w^2 c}{eHA} . \qquad (23)$$

In a torus, H varies as 1/R, where R is the distance from

the center of the torus. Adding these two drifts we have

$$v_y = \frac{m\,c}{eH}\left(w^2+\tfrac{1}{2}u^2\right)\frac{1}{A} \quad . \qquad (24)$$

The quantity eH/m c is simply the angular frequency of gyration in the magnetic field, and equals about 10^8 for deuterons and 10^{11} for electrons, if H is 2 x 10^4 gauss.

For a Maxwellian distribution of velocities we have

$$v_y = \frac{2\,k\,T\,c}{e\,H\,A} \quad . \qquad (25)$$

If A is 10^3 centimeters v_y is about 10^5 cm/sec. In one tour around the torus the average particle would experience a total lateral drift of about $2\pi\rho$, or some 6 centimeters for deuterons.

Actually these drifts would not normally materialize, since they would produce a large separation of charge, The resultant electrostatic field would then produce a drift of both electrons and positive ions to the outer side of the torus, and all the material in the Stellarator would strike the outer wall before one circuit of the torus had been completed, To prevent this result it would be necessary to feed in positive ions and electrons at the top and bottom of the Stellarator, thus allowing the vertical drifts to develop without producing an electrostatic charge. If the tube radius were 50 centimeters, the particles would drift across the tube in 10^{-3} seconds,

somewhat less than the time between collisions. The loss of kinetic energy at the top and bottom of the tube would exceed P_W found in equation (21) by a factor of 10^4, and would much exceed P_N. The recoil nuclei from the nuclear reactions would leave even more rapidly. Evidently a toroidal Stellarator is not practical.

To permit the reaction to continue a basic modification of the shape of the tube is proposed. The tub8 may be bent into the shape of a figure eight or pretzel, as shown in Figure 1. The two end loops need not be in the same plane but their planes should be parellel. This modification consists essentially of two tari, with magnetic fields in opposite directions, linked together to form a pretzel. It is readily seen that the drift velocity of an ion going around one end of the pretzel will be in the opposite direction from the drift velocity for the same ion going around the other end. Thus a positive ion will go up about 3 cm going around one end and down 3 cm going around the other end. For the electrons the drifts are less by a factor of 60. Thus we see that for particles which have an appreciable velocity along the lines of force the drifts will cancel out in one circuit of the tube,

Those ions whose velocity component along the lines of force is mall will pass slowly around one end of the tube, and for these the drift velocity may be serious.

In particular, any ion which takes longer than 10^{-3} seconds to pass around one end will be driven against the top or bottom of the tube. If the tube is bent in a circle of radius 300 centimeters, all particles whose velocities along the lines of force are less than 10^{6} cm/sec will drift out. Only about one percent of the ions will have such a low velocity in one direction, and on the average a particular ion will possess such a low velocity for only a small fraction of the relaxation time, 3×10^{-3} sec. Moreover, the presence of a radial electrical field will destroy the systematic drift entirely, producing instead a spiralling motion of each ion about the tube axis. Some additional electrostatic fields will tend to develop, but along each line of force these additional fields will have opposite signs in the two ends of the pretzel, and will therefore be readily neutralized. Either System A or System B could be constructed in this particular pretzel-like shape.

A number of individual details remains to be discussed. Different methods could be used for starting a Stellarator. For System A, one method would be to start the current in the magnet coils, and to excite by induction a glow discharge in the tube to bring the gas up to a high temperature. This could perhaps be done most easily at a very low density, and then the density

gradually increased after high temperatures had been reached. The Stellarator could be readily stopped by cutting off the supply of reacting material; in the deuterium-tritium Stellarator discussed above, half the positive ions combine to form He^4 within about a hundred seconds.

The proposed Stellarator, System A, would contain about 0.03 grams of tritium, and would consume about ten kilograms of tritium a year in steady operation, liberating about 150,000 kilowatts of nuclear power in steady operation. Only a fraction of this power would be available for other purposes, when the thermodynamic inefficiency of heat engines and the power required for the magnet are taken into account. In principle, all the tritium consumed could be regenerated by neutron capture in lithium, placed between the wall of the tube and the coils of the magnet. Heavy water could be used for slowing down the neutrons, cooling the lithium, and for subsequent generation of electrical power with the heat received. The characteristics of System B would be similar, except that a larger fraction of the nuclear power liberated would be available for other purposes.

A deuteron-deuteron Stellarator would produce as much power, or more, and would generate tritium. Since the ratio of P_N to P_H in System A varies as $a^2H^2\alpha\sigma$, the reduced nuclear cross section would require increasing

a or H by considerable factors, even if liquid air temperatures were used to raise σ. An increase 'of a or H will bring the power back to nearlg'the same value as in the deuterium-tritium Stellarator, but an increase in a will require an increase in the total length of the tube. A tube 400 meters in length, with a power output of 30 kilowatts per centimeter, would generate about a million kilowatts of power, would produce ,a tenth of a ton of tritium per year, and about the same quantity of He^3; this latter isotope, like H^3, could also be used in smaller Stellaratore for power generation, and would have the considerable advantage that no neutrons or gamma rays would be produced. System B would probably be more suitable for the deuteron-deuteron Stellarator than System A, but would presumably also require a relatively large installation,

V. Summary of Proposed Stellarator Design

The proposed thermonuclear Stellarator, designed to operate with a mixture of deuterium and tritium, would consist primarily of a cylindrical tube, about. 50 centimeters in radius and some 40 to 50 meters in length, Dent around to fom a figure sight, or pretzel - see Figure 1, Immediately surrounding this tube would be a shell of heavy water to slow down tho neutrons and absorb the

Stellarator energy. Outside this would be a shell of water-cooled lithium. in which the neutrons would be absorbed and tritium generated. Outside all this would be a coil of copper wire, with'an outer radiue twice the inner radius, through which a strong current would be passed, creating a magnetic field of some 20,000 gauss, parallel at each point to the axis of the curved tube, This coil would also need water cooling.

Inside the tube would be a mixture of gaseous deuterium and tritium, with a density of about 2 x 10^{-9} grams per cubic centimeter. This gas would be excited initially by a glow discharge, or some other means, to a temperature of about 100,000,000 degrees Kelvin, and would then maintain itself at this high temperature. To prevent the formation of a low-temperature region near the tube walle, the gas approaching the wall would be taken outof the tube before it reached the wall, and very little gas would be cooled by collision8 with the wall. This result, would be achieved by the use of several pairs of auxiliary tubes, each with its own magnetic field, placed at right angles to the primary field in a gap in this field, as shown in Figure 2. The gas moving along these auxiliary tubes, in diverging magnetic fields, would then be stopped at walls, compressed, and shot back into the main tube in the form of jets. A certain fraction of the gas, containing He4, would be taken away in this process, and fresh deuterium-tritium mixture added.

A modification to this sytem would use the external coils only for generating a momentary magnetic field, which would then be retained within the tuba for several minutes by the high conductivity of the very hot ionized gas,, In this case the auxilary tubes would not be needed for taking gas out of the chamber, as neutral hydrogen and helium at a moderately low temperature could be taken out through any opening in the main tube; it would still be advantageous, however, to shoot the fresh deuterium-tritium mixture back into the tubs in the form of high-speed jets.

This proposed Stellarator would liberate some hundred thousand kilowatts of nuclear power, and would consume ten kilograms of tritium a year, which would be regenerated, however, in the lithium. A Stellarator with an external magnetic field and designed to operate with pure deuterium would be ten times larger, or would require magnetic fields of 100,000 gauss, or some partial, combination of these unless liquid air cooling could be used for the magnet coils, in which case smaller increases would be required. If ordinary water cooling were used, the nuclear power released would be in the region 100,000 to 1,000,000 kilowatts and between 10 and 100 kilograms of tritium would be produced per year. A Stellarator whose magnetic field was produced with the reacting gas could probably maintain reactions in pure deuterium on a somewhat smaller scale if' desired.

Figure 1

Diagram of Stellarator Tube

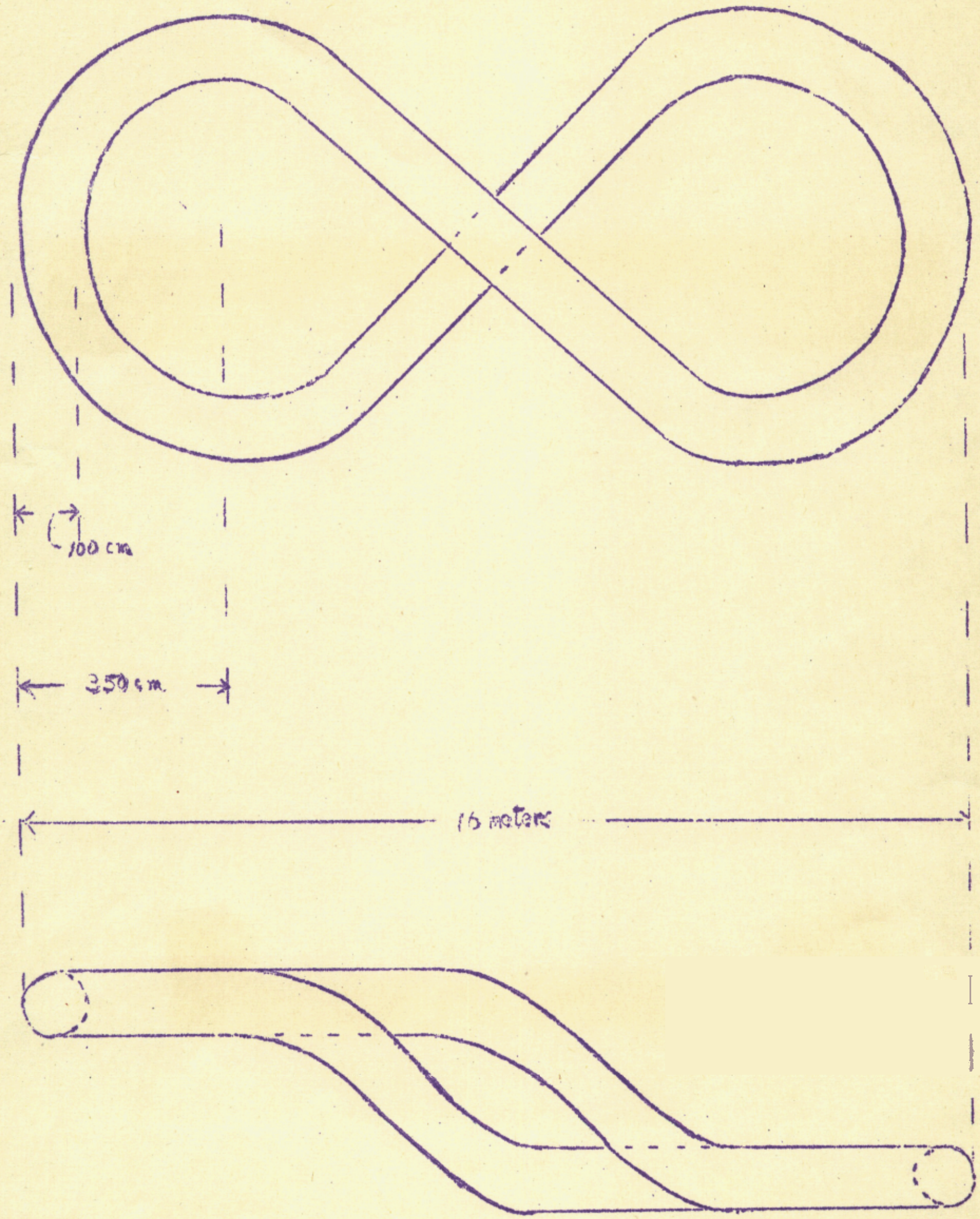
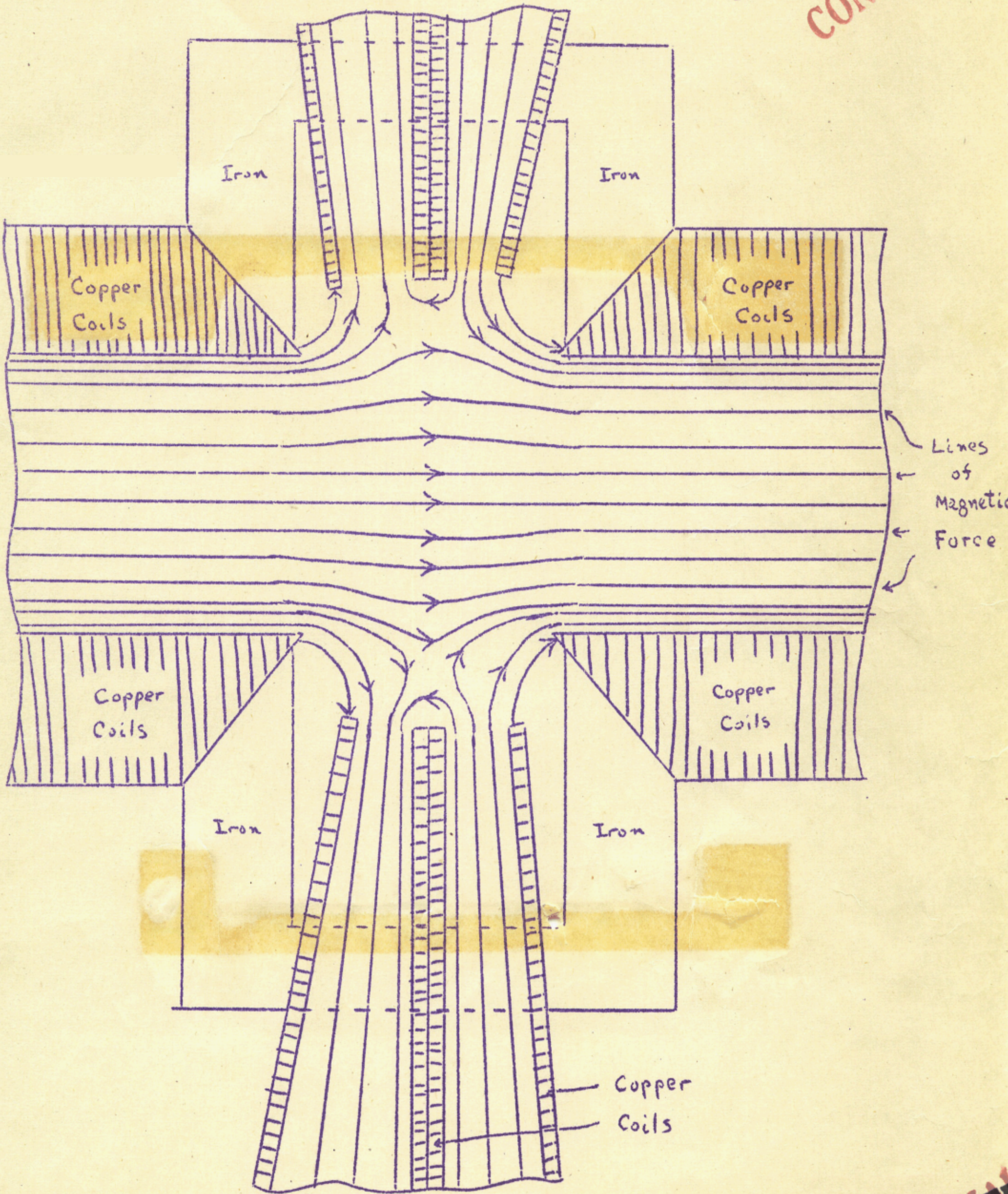


Figure 2

Proposed Arrangement for Extracting Gas



Proportions are schematic only